Understanding relationships between numbers can save you time when making calculations. Previously, you worked with factors and multiples of various numbers, and you determined which numbers are prime and composite by using the Sieve of Eratosthenes. By doing so, you determined what natural numbers are divisible by other natural numbers.

In this lesson, you will consider patterns for numbers that are divisible by 2, 3, 4, 5, 6, 9, and 10. What type of patterns do you think exist between these numbers? Why do you think 1 is not a part of this list?
Problem 1
Students explore the divisibility of numbers by 2, 5 and 10. They will list multiples of given numbers and notice all multiples of 2 are even numbers, all multiples of 5 have a last digit that ends in a 0 or 5, and all multiples of 10 are also multiples of both 2 and 5. Students will write divisibility rules for 2, 5, and 10.

Grouping
Have students complete Questions 1 through 3 with a partner. Then share the responses as a class.

Share Phase, Questions 1 through 3
- All numbers are divisible by what number?
- Are all numbers divisible by some other number?
- Can you think of a number that is not divisible by any other number?
- What number is the multiplicative identity?
- Are there any shortcuts you know for checking for divisibility?
- If you listed 20 multiples for each number, would the same pattern emerge?
- Can a divisibility rule be used on all numbers?
- If a number is divisible by 5 is it also divisible by 10?

Problem 1 Exploring Two, Five, and Ten

1. List 10 multiples for each number.
   - Multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20
   - Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50
   - Multiples of 10: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100

2. What do you notice?
   - All multiples of 2 are even and end in 0, 2, 4, 6, or 8.
   - All multiples of 5 end in a 0 or a 5.
   - All multiples of 10 end in a 0. Also, all multiples of 10 are also multiples of 2 and 5.

Divisibility rules are tests for determining whether one whole number is divisible by another. A divisibility rule must work for every number.

3. Write a divisibility rule for 2, 5, and 10. Then, show an example that follows your rule.

<table>
<thead>
<tr>
<th>A natural number is divisible by</th>
<th>...if</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>the ones digit is 0, 2, 4, 6, or 8; the number is even.</td>
<td>186 is divisible by 2 because the ones digit is 6.</td>
</tr>
<tr>
<td>5</td>
<td>the ones digit is either 0 or 5.</td>
<td>145 is divisible by 5 because the ones digit is 5.</td>
</tr>
<tr>
<td>10</td>
<td>the ones digit is 0.</td>
<td>630 is divisible by 10 because the ones digit is a 0.</td>
</tr>
</tbody>
</table>

- If a number is divisible by 10 is it also divisible by 5?
- If a number is divisible by both 2 and 5, why must it be divisible by 10?
**Problem 2**

Students explore the divisibility of numbers by 3, and 6. They will begin by using a list of numbers that are divisible by 3 and determine which of the given numbers are divisible by 2, 5, and 10. After analyzing given numbers, students will look for patterns and write divisibility rules for 3, and 6. They then test each divisibility rule using a calculator.

**Materials**

Calculator

**Grouping**

Have students complete Questions 1 through 5 with a partner. Then share the responses as a class.

**Share Phase, Questions 1 and 2**

Consider all of the digits of the number, what operation can you perform that may help you determine a pattern in order to write a rule for the divisibility of a number by 3?

<table>
<thead>
<tr>
<th>Number</th>
<th>Divisible by 2</th>
<th>Divisible by 3</th>
<th>Divisible by 5</th>
<th>Divisible by 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>✔️</td>
<td>✔️</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>1071</td>
<td></td>
<td>✔️</td>
<td></td>
<td></td>
</tr>
<tr>
<td>882</td>
<td>✔️</td>
<td>✔️</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1230</td>
<td>✔️</td>
<td>✔️</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>285</td>
<td></td>
<td></td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>3762</td>
<td>✔️</td>
<td>✔️</td>
<td></td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>✔️</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2784</td>
<td>✔️</td>
<td></td>
<td>✔️</td>
<td></td>
</tr>
<tr>
<td>3562</td>
<td>✔️</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>111</td>
<td></td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
</tbody>
</table>

1. Place a check in the appropriate column for each number that is divisible by 2, 5, or 10.

2. Analyze each number that is divisible by 3. Then, write a rule in the table shown to indicate when a number is divisible by 3. (Hint: Consider the sum of the digits of the number.)

<table>
<thead>
<tr>
<th>A number is divisible by</th>
<th>...if</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>the sum of the digits in the number is divisible by 3.</td>
<td>168 is divisible by 3 because the sum of the digits is 15 (1 + 6 + 8), and 15 is divisible by 3.</td>
</tr>
</tbody>
</table>
Share Phase, Questions 3 through 5

- If a number is divisible by both 2 and 3, why must it be divisible by 6?
- If a number is divisible by 3, is it also divisible by 6?
- If a number is divisible by 6, is it also divisible by 3?
- If a number is divisible by 6, why must it be divisible by 3?

3. Circle numbers you think are divisible by 6 in the table you completed in Question 1. Explain your reasoning.

   The numbers I think are divisible by 6 are 300, 882, 1230, 3762, 42, 2784, and 3582. These numbers are divisible by both 2 and 3, which are both factors of 6. So, I think that these numbers are also divisible by 6.

4. Analyze each number you circled that you think is divisible by 6. Write a rule to indicate when a number is divisible by 6 in the table shown.

<table>
<thead>
<tr>
<th>A number is divisible by</th>
<th>...if</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>the number is divisible by both 2 and 3.</td>
<td>264 is divisible by 6 because it is divisible by 2 and divisible by 3.</td>
</tr>
</tbody>
</table>

5. Test the divisibility rules you wrote to indicate if a number is divisible by 3 or 6 by writing several three- or four-digit numbers that you think are divisible by 3 or 6. Then, use your calculator to determine if the numbers you wrote are divisible by 3 or 6.

   Answers will vary.
**Problem 3**

Students explore the divisibility of numbers by 9. They begin by using a list of numbers that are divisible by 9 and determine which of the given numbers are divisible by 2, 3, 5, 6, and 10. After analyzing given numbers, students look for patterns and write a divisibility rule for 9. They then test the divisibility rule using a calculator.

**Grouping**

Have students complete Questions 1 through 3 with a partner. Then share the responses as a class.

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### Problem 3 Exploring Nine

1. Place a check in the appropriate column for each number that is divisible by 2, 3, 5, 6, or 10. The column for Divisible by 9 is completed for you.

<table>
<thead>
<tr>
<th>Number</th>
<th>Divisible by 2</th>
<th>Divisible by 3</th>
<th>Divisible by 5</th>
<th>Divisible by 6</th>
<th>Divisible by 9</th>
<th>Divisible by 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>3240</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>1458</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>18,225</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>2025</td>
<td>✓</td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>7878</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3477</td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2565</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>285</td>
<td>✓</td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>600</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

2. Analyze the numbers shown in the list. Write a rule to indicate when a number is divisible by 9. (Hint: Use the same clue you were given when exploring the divisibility rule for 3.)

<table>
<thead>
<tr>
<th>A number is divisible by</th>
<th>...if</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>The sum of the digits of the number is divisible by 9.</td>
<td>279 is divisible by 9 because the sum of the digits is 18 (2 + 7 + 9), and 18 is divisible by 9.</td>
</tr>
</tbody>
</table>
Share Phase, Questions 1 through 3

- If a number is divisible by 3, is it also divisible by 9?
- If a number is divisible by 9, is it also divisible by 3?
- If a number is divisible by 6, is it also divisible by 9?
- If a number is divisible by 9, is it also divisible by 6?
- If a number is divisible by 9, why must it be divisible by 3?

Problem 4 Exploring Four

Each number listed in the table is divisible by 4.

<table>
<thead>
<tr>
<th>Numbers Divisible by 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>116</td>
</tr>
<tr>
<td>1436</td>
</tr>
<tr>
<td>228</td>
</tr>
<tr>
<td>2524</td>
</tr>
<tr>
<td>41,032</td>
</tr>
</tbody>
</table>

1. What pattern do you notice about each number? (Hint: Look at the number formed by the last two digits in each number.)
   - Each number is an even number, or each number is divisible by 2.
   - The last two digits of each number also form a number that is divisible by 4.

2. Write a rule to tell when a number is divisible by 4.

<table>
<thead>
<tr>
<th>A number is divisible by</th>
<th>...If</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>the number formed by the last two digits is divisible by 4.</td>
<td>316 is divisible by 4 because 16 is divisible by 4.</td>
</tr>
</tbody>
</table>

3. Test the divisibility rule you wrote to indicate if a number is divisible by 9 by writing several four- or five-digit numbers that you think are divisible by 9. Then, use your calculator to determine if the numbers you wrote are divisible by 9.
   - Answers will vary.

Grouping

Have students complete Questions 1 through 3 with a partner. Then share the responses as a class.
1.4 Investigating Divisibility Rules

Share Phase, Questions 1 through 3

- If a number is divisible by 2, is it also divisible by 4?
- If a number is divisible by 4, is it also divisible by 2?
- If a number is divisible by 4, is it also divisible by 8?
- If a number is divisible by 8, is it also divisible by 4?
- If a number is divisible by 4, why must it be divisible by 2?

Problem 5
Students use the divisibility rules they have created to test the divisibility of several numbers and explain their reasoning. They create numbers that are divisible by given numbers. Given a series of clues and using the divisibility rules, students identify a mystery number.

Grouping

Have students complete Questions 1 through 5 with a partner. Then share the responses as a class.

Share Phase, Questions 1 and 2

- If a number is divisible by 3, what numbers could be the last digit?
- If a number is divisible by 3, what numbers could not be the last digit?

Problem 5 It's a Mystery

1. Determine if each number is divisible by 3 using your divisibility rule. Explain your reasoning.
   a. 597
      Yes. The number 597 is divisible by 3 because $5 + 9 + 7 = 21$, and 21 is divisible by 3.
   b. 2109
      Yes. The number 2109 is divisible by 3 because $2 + 1 + 0 + 9 = 12$, and 12 is divisible by 3.
   c. 83,594
      No. The number 83,594 is not divisible by 3 because $8 + 3 + 5 + 9 + 4 = 29$, and 29 is not divisible by 3.

2. Determine if each number is divisible by 9 using your divisibility rule. Explain your reasoning.
   a. 748
      No. The number 748 is not divisible by 9 because $7 + 4 + 8 = 19$, and 19 is not divisible by 9.
   b. 5814
      Yes. The number 5814 is divisible by 9 because $5 + 8 + 1 + 4 = 18$, and 18 is divisible by 9.
   c. 43,695
      Yes. The number 43,695 is divisible by 9 because $4 + 3 + 6 + 9 + 5 = 27$, and 27 is divisible by 9.

- If a number is divisible by 9, what numbers could be the last digit?
- If a number is divisible by 9, what numbers could not be the last digit?
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Share Phase, Questions 3 through 5

• If a number is a two digit number, can you use any divisibility rules to eliminate a factor?
• What multiples of 5 do not have a last digit that is a 5?
• What are some numbers that are both a multiple of 5 and divisible by 3?

3. Fill in the missing digit for each number to make the sentence true.
   a. The number 10,5__2 is divisible by 6.
      The possible values are 1, 4, or 7.
   b. The number 505__ is divisible by 4.
      The possible values are 2 or 6.
   c. The number 133,0__5 is divisible by 9.
      The value is 6.

4. Rasheed is thinking of a mystery number. Use the following clues to determine his number. Explain how you used each clue to determine Rasheed’s number.
   Clue 1: My number is a two-digit number.
   Clue 2: My number is a multiple of 5, but does not end in a 5.
   Clue 3: My number is less than 60.
   Clue 4: My number is divisible by 3.
   Rasheed’s number is 30.
   Clue 1: The number is between 10 and 99 inclusive.
   Clue 2: Since the number is a multiple of 5, but not ending in 5, the number must end in a zero. The number could be 10, 20, 30, 40, 50, 60, 70, 80, or 90.
   Clue 3: Since the number is less than 60, it could be 10, 20, 30, 40, or 50.
   Clue 4: Since the number is divisible by 3, the number can only be 30.

5. Think of your own mystery number, and create clues using what you know about factors, multiples, and the divisibility rules. Give your clues to your partner. See if your partner can determine your mystery number!
   Answers will vary based on correct use of divisibility rules, factors, and multiples.
**Talk the Talk**

The divisibility rules for the numbers 2, 3, 4, 5, 6, 8, 9, and 10, are summarized.

**Grouping**

Ask a student to read the summary for the divisibility rules and the text in the speech bubble aloud. Then have the students complete Question 1 independently and share the responses as a class.

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**Talk the Talk**

Divisibility rules are tests for determining whether one number is divisible by another number.

A number is divisible by:

- 2 if the number is even.
- 3 if the sum of the digits is divisible by 3.
- 4 if the number formed by the last two digits is divisible by 4.
- 5 if the number ends in a 0 or a 5.
- 6 if the number is divisible by both 2 and 3.
- 9 if the sum of the digits is divisible by 9.
- 10 if the last digit is 0.

1. Determine if each number is divisible by 8 using the divisibility rule.
   a. 75,024
      Yes. The last three digits are divisible by 8.
   b. 1466
      No. The last three digits are not divisible by 8.
   c. 19,729
      No. The last three digits are not divisible by 8.
   d. 1968
      Yes. The last three digits are divisible by 8.

Be prepared to share your solutions and methods.